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HYDROMAGNETIC FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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An exact solution of the MHD Stokes problem for the flow of an electrically conducting, incompressible, viscous fluid past an impulsively started infinite vertical plate in the presence of variable temperature and mass diffusion is obtained. The dimensionless governing equations are solved using the Laplace-transform technique. The plate temperature and the concentration level near the plate increase linearly with time. The solutions for the velocity and skin friction are obtained for different magnetic field parameters and multiple buoyancy effects for aiding and opposing flows. It is observed that the velocity decreases in the presence of a magnetic field as compared to its absence and that the skin friction increases in the presence of aiding flows and decreases with opposing flows.

Introduction. The influence of a magnetic field on the viscous incompressible flow of an electrically conducting fluid is of importance in many applications, such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, the paper industry, the textile industry, in different geophysical cases, etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subjected to a magnetic field.

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, and magnetic suppression of molten semiconducting materials. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets, chemical synthesis, MHD power generators, etc. In the field of power generation, MHD is attracting considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Boundary-layer flow on moving horizontal surfaces was studied by Sakiadis [1]. The effect of the presence of a foreign mass on the free-convection flow past a semi-infinite vertical plate was first considered by Gebhart and Pera [2]. Kumari and Nath [3] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface with an applied magnetic field, when the external stream or the stretching surface was set into impulsive motion from the rest. Vajravelu [4] considered the exact solution for hydrodynamic boundary-layer flow and heat transfer over a continuous, moving, vertical flat surface with uniform suction and internal heat generation/absorption. In all these studies, the authors have taken the continuous moving surface to be oriented in the horizontal direction. The effects of a transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al. for the cases where this plate is isothermal [5] and where it is characterized by variable temperature [6]. The dimensionless governing equations were solved using the Laplace-transform technique.

It is proposed to study the flow past an impulsively started infinite vertical plate in the presence of variable temperature and mass diffusion with a transverse applied magnetic field. The governing equations are solved by the Laplace-transform technique. In this paper, the buoyancy-ratio parameter is introduced. When the temperature is high enough, buoyancy effects also generate a significant flow which aids or opposes this induced flow. There are many

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situations in the manufacturing industry, especially in metal forming and heat treatment, in which one encounters energy transfer to the surroundings from a moving material. The effect of magnetic field and buoyancy-ratio parameters for aiding and opposing flows is studied.

Mathematical Formulation. The hydromagnetic flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate with variable mass diffusion is studied. Here the x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken to be normal to the plate. Initially, the plate and fluid are at the same temperature and the concentration is constant. At the time $t' > 0$, the plate is subjected to an impulsive motion in the vertical direction against a gravitational field with constant velocity u_0. The plate temperature and the concentration level near the plate increase linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normally to the plate. The induced magnetic field and viscous dissipation are assumed to be negligible. Then by the usual Boussinesq approximation, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta\,(T' - T'_\infty) + g\beta^*\,(C' - C'_\infty) + \nu\,\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}\,u' , \tag{1}$$

$$\rho C_p\,\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial y'^2} , \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\,\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

with initial and boundary conditions

$$t' \le 0 : \quad u' = 0 , \quad T' = T'_\infty , \quad C' = C'_\infty \quad \text{for all } y' ;$$

$$t' > 0 : \quad u' = u_0 , \quad T' = T'_\infty + (T'_w - T'_\infty)\,At' , \quad C' = C'_\infty + (C'_w - C'_\infty)\,At' \quad \text{at } y' = 0 ,$$

$$u' = 0 , \quad T' \to T'_\infty , \quad C' \to C'_\infty \quad \text{as } y' \to \infty , \tag{4}$$

where $A = u_0^2/\nu$.

Introduction of the dimensionless quantities

$$u = \frac{u'}{u_0 \mathrm{Gr}} , \quad t = \frac{t' u_0^2}{\nu} , \quad y = \frac{y' u_0}{\nu} , \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} ,$$

$$\mathrm{Gr} = \frac{g\beta\nu\,(T'_w - T'_\infty)}{u_0^3} , \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty} , \quad \mathrm{Gm} = \frac{\nu g\beta^*\,(C'_w - C'_\infty)}{u_0^3} , \tag{5}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k} , \quad \mathrm{Sc} = \frac{\nu}{D} , \quad N = \frac{\mathrm{Gm}}{\mathrm{Gr}} , \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$$

in Eqs. (1)–(4) leads to

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} - Mu \,, \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \,, \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\text{Sc}}\frac{\partial^2 C}{\partial y^2} \,. \tag{8}$$

The initial and boundary conditions in dimensionless form are

$$t \le 0: \quad u = 0\,, \quad \theta = 0\,, \quad C = 0 \ \text{ for all } y\,;$$

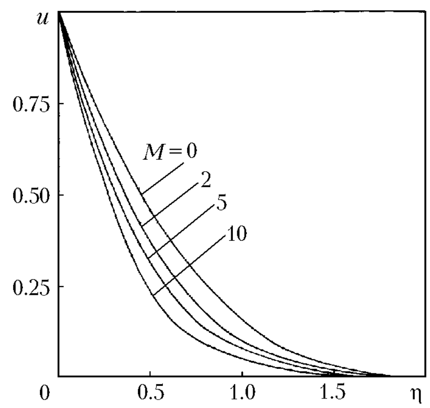
$$t > 0: \quad u = \frac{1}{\text{Gr}}\,, \quad \theta = t\,, \quad C = t \ \text{ at } \ y = 0\,,$$

$$u = 0\,, \quad \theta \to 0\,, \quad c \to 0 \ \text{ as } \ y \to \infty\,. \tag{9}$$

Equations (6)–(8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\eta^2 \Pr)\,\text{erfc}\,(\eta\sqrt{\Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}}\,\exp\,(-\eta^2 \Pr)\right], \tag{10}$$

$$\begin{aligned}
u = {} & \frac{1}{2}\left(\frac{1}{\text{Gr}} - \frac{1}{aM} - \frac{N}{bM} - \frac{(1+N)\,t}{M}\right)[\exp\,(2\eta\sqrt{Mt})\,\text{erfc}\,(\eta + \sqrt{Mt}) + \exp\,(-2\eta\sqrt{Mt})\,\text{erfc}\,(\eta - \sqrt{Mt})] \\
& + \frac{\eta\,(1+N)\sqrt{t}}{2M\sqrt{M}}\,[\exp\,(-2\eta\sqrt{Mt})\,\text{erfc}\,(\eta - \sqrt{Mt}) - \exp\,(2\eta\sqrt{Mt})\,\text{erfc}\,(\eta + \sqrt{Mt}) \\
& + \frac{1}{aM}\,\text{erfc}\,(\eta\sqrt{\Pr}) + \frac{N}{bM}\,\text{erfc}\,(\eta\sqrt{\text{Sc}}) + \frac{\exp\,(at)}{2aM}\,[\exp\,(2\eta\sqrt{(M+a)\,t})\,\text{erfc}\,(\eta + \sqrt{(M+a)\,t}) \\
& + \exp\,(-2\eta\sqrt{(M+a)\,t})\,\text{erfc}\,(\eta - \sqrt{(M+a)\,t})] + \frac{N\exp\,(bt)}{2bM}\,[\exp\,(2\eta\sqrt{(M+b)\,t})\,\text{erfc}\,(\eta + \sqrt{(M+b)\,t}) \\
& + \exp\,(-2\eta\sqrt{(M+b)\,t})\,\text{erfc}\,(\eta - \sqrt{(M+b)\,t})] + \frac{t}{M}\left[(1 + 2\eta^2\Pr)\,\text{erfc}\,(\eta\sqrt{\Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}}\,\exp\,(-\eta^2\text{Sc})\right] \\
& + \frac{Nt}{M}\left[(1 + 2\eta^2\text{Sc})\,\text{erfc}\,(\eta\sqrt{\text{Sc}}) - 2\eta\sqrt{\frac{\text{Sc}}{\pi}}\,\exp\,(-\eta^2\text{Sc})\right] - \frac{\exp\,(at)}{2aM}\,[\exp\,(2\eta\sqrt{a\Pr t})\,\text{erfc}\,(\eta\sqrt{\Pr} + \sqrt{at}) \\
& + \exp\,(-2\eta\sqrt{a\Pr t})\,\text{erfc}\,(\eta\sqrt{\Pr} - \sqrt{at})] - \frac{N\exp\,(bt)}{2bM}\,[\exp\,(2\eta\sqrt{b\text{Sc}t})\,\text{erfc}\,(\eta\sqrt{\text{Sc}} + \sqrt{bt}) \\
& + \exp\,(-2\eta\sqrt{b\text{Sc}t})\,\text{erfc}\,(\eta\sqrt{\text{Sc}} - \sqrt{bt})]\,,
\end{aligned} \tag{11}$$



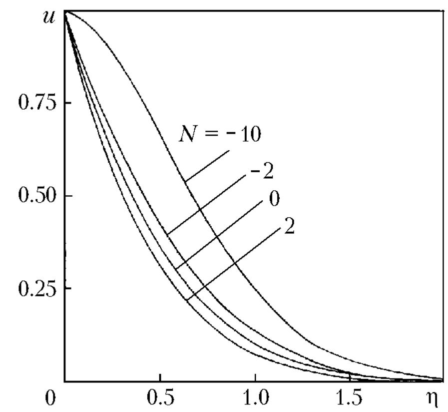


Fig. 1. Velocity profiles for different M at Pr = 7, N = 0.2, Sc = 2.01, Gr = 1, and t = 0.2.

Fig. 2. Velocity profiles for different N at Pr = 7, Sc = 2.01, M = 2, and t = 0.2.

$$C = t\left[(1 + 2\eta^2 \mathrm{Sc})\,\mathrm{erfc}\,(\eta\sqrt{\mathrm{Sc}}) - 2\eta\sqrt{\frac{\mathrm{Sc}}{\pi}}\,\exp\,(-\eta^2 \mathrm{Sc})\right], \tag{12}$$

where $a = \dfrac{M}{\mathrm{Pr} - 1}$, $b = \dfrac{M}{\mathrm{Sc} - 1}$, and $\eta = \dfrac{y}{2\sqrt{t}}$.

Results and Discussion. The numerical values of the velocity and skin friction are computed at Gr = 1, Sc = 2.01, and Pr = 7 for different times and magnetic field and buoyancy-ratio parameters. The purpose of the calculations given here is to assess the effects of the parameters M and N upon the nature of the flow and transport.

The velocity profiles for different values of the magnetic field parameter are shown in Fig. 1. It is observed that the velocity in the presence of a magnetic field is smaller than in its absence and that the greater the magnetic field parameter, the smaller the velocity. The effect of the buoyancy-ratio parameter for both aiding and opposing flows is shown in Fig. 2. In this case, the velocity decreases in the presence of aiding flows, whereas it increases in the opposing flows.

From the velocity field, we now study the skin friction. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0}. \tag{13}$$

Hence, from Eqs. (11) and (13), the wall shear stress in the presence of a magnetic field is as follows:

$$\tau = \frac{1}{\sqrt{\pi t}}\Bigg[\left(-\frac{1}{\mathrm{Gr}} + \frac{1}{aM} + \frac{N}{bM} + \frac{(1+N)\,t}{M}\right)(\sqrt{M\pi t}\,\mathrm{erfc}\,(\sqrt{Mt}) - 1) + \frac{(1+N)\sqrt{\pi t}}{2M\sqrt{M}}\,\mathrm{erfc}\,(\sqrt{Mt})$$

$$+ \frac{(1+2t)\sqrt{\mathrm{Pr}}}{aM} + \frac{(1+2t)\,N\sqrt{\mathrm{Sc}}}{bM} + \frac{\exp\,(at)}{M}\,(1 - \sqrt{\mathrm{Pr}} - \sqrt{(M+a)\,\pi t}\,\mathrm{erfc}\,(\sqrt{(M+a)\,\pi t}$$

$$+ \sqrt{\mathrm{Pr}\pi at}\,\mathrm{erfc}\,\sqrt{at}) + \frac{N\exp\,(bt)}{bM}\,(1 - \sqrt{\mathrm{Sc}} - \sqrt{(M+b)\,\pi t}\,\mathrm{erfc}\,\,(\sqrt{(M+b)\,\pi t} + \sqrt{\mathrm{Sc}\pi bt}\,\mathrm{erfc}\,\sqrt{bt})\Bigg]. \tag{14}$$

The numerical values of the skin friction τ are presented in Table 1 for Gr = 2, Pr = 7, and Sc = 2.01. It is observed from this table that skin friction decreases with increasing values of the magnetic field parameter and time.

TABLE 1. Values of Skin Friction τ

M	N	t	τ
2	2	0.2	6.9114
5	2	0.2	1.4070
10	2	0.2	0.7215
2	0	0.2	5.4659
2	–5	0.2	1.8523
2	–10	0.2	–1.7614
2	2	0.4	6.800

This shows that the wall shear stress increases with decreasing magnetic field parameter and time. It is also seen that the skin friction increases in the presence of aiding flows and decreases with opposing flows.

Conclusions. An analysis is performed to study the flow past an impulsively started infinite vertical plate with variable temperature and mass diffusion in the presence of a transverse applied magnetic field. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effects of magnetic field and buoyancy-ratio parameters are studied. The conclusions are as follows:

1. The velocity in the presence of a transverse field is smaller than in its absence.
2. The velocity increases in the presence of opposing flows ($N < 0$) and decreases with aiding flows ($N > 0$).
3. The skin friction increases in the presence of aiding flows and decreases with opposing flows.

NOTATION

A, constant; B_0, external magnetic field strength; C', species concentration in the fluid; C'_w, concentration near the plate; C'_∞, concentration in the fluid far away from the plate; C, dimensionless concentration; C_p, specific heat at constant pressure; D, diffusivity; g, acceleration due to gravity; Gr, thermal Grashof number; Gm, mass Grashof number; k, thermal conductivity of the fluid; M, magnetic field parameter; N, buoyancy-ratio parameter; Pr, Prandtl number; Sc, Schmidt number; T', temperature of the fluid near the plate; T'_w, temperature of the plate; T'_∞, temperature of the fluid far away from the plate; t', time; t, dimensionless time; u', velocity of the fluid in the x'-direction; u_0, velocity of the plate; u, dimensionless velocity; y', coordinate normal to the plate; y, dimensionless coordinate normal to the plate; β, volumetric coefficient of thermal expansion; β^*, volumetric coefficient of expansion with concentration; μ, coefficient of viscosity; ν, kinematic viscosity; ρ, density of the fluid; σ, electric conductivity; τ', skin friction; τ, dimensionless skin friction; θ, dimensionless temperature; η, similarity parameter.

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